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The Development of Mathematical Achievement in Analytic Geometry of Grade-12 Students through GeoGebra Activities

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ABSTRACT

This research provides the instructional exploration of analytic geometry pattern based on van Hiele thinking pattern, and the potential of GeoGebra effect on experimental group along with its nested group (high and low achievers) in comparison with control group in analytic geometry. To investigate the significant effect of GeoGebra, the two match groups were constructed on their previous grade-11 mathematics records with almost equal statistical background and with the same compatibility in the biological age. Further, six-week experiments of 22 lessons were prepared and two teaching methods (tradition vs DGS aided instructions) were tested. Three hypotheses were carried out i.e. Treatment does not significantly affect, the two groups in mathematical achievement mean scores and, the higher and low achievers of the two groups in mathematical achievement mean scores. To measure the treatment effect, t-test was used by SPSS. Analyses of the research revealed that experimental group performed well, while; GeoGebra was influential in favor of low achievers in comparison to control low achievers.

Keywords: mathematical achievement, GeoGebra, diverse achievers

INTRODUCTION

In our education system, Mathematics is the key and a tough subject in both teaching and learning. The teacher's role is paramount for implementing the curriculum of Mathematics. While, its effectiveness is commonly measured through the mathematical achievement of students, and teachers are mainly considered responsible for the improvement of this key indicator. In addition, it is accepted that technology positively affects the class room instructions but in Pakistan its use is very poor (Iqbal, Shawana, & Saeed, 2013). In higher secondary mathematics, students face difficulty in conceptualizing most of the concepts. GeoGebra, which is a free software tool for plane analytic geometry understanding and it is being used in most of the country to support abstract concept in a concrete way. Despite this, its application in teaching Mathematics has not been acknowledge in Pakistan. Particularly, to make this package effective in favor of students, teacher's role is essential in explaining and exploring the mathematical concepts by using interactive and dynamic applets in systematic ways (Ljajko, 2013).

THE NATURE OF ANALYTIC GEOMETRY AND PRE-REQUISITE OF ITS TEACHING

The French mathematician and philosopher Rene Descartes (1595-1650) had used algebraic method in solving geometry problem, which caused the birth of analytic geometry (KPK text Book for 2nd year). It is the potential of this subject that the geometric relationship of the object can be seen and transferred to the world of abstract

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Contribution of this paper to the literature

- This research provides the structural issues of analytic geometry and the prerequisite for its teaching-learning.
- This research explores Van Hiele's thinking pattern in analytic geometry.
- the paper critically describes different dynamic features of Geogebra in developing the achievement of students in analytic geometry.

arithmetic relationship (Waismann, 1951). The semiotic system of representation is the main essence of this discipline: algebraic representation and geometric representation (Hesselbart, 2007). Further, both representations consist of structure of generalized points in gestalt way. And every structure deals with variables, parameters and constants with interconnected relationship. Because of its abstract nature along with dual representation, students have a lot of misconceptions in understanding most of the concepts of this subject.

The fundamental of analytic geometry that is to show the relation between two or more variables graphically and, the change in one variable will cause the corresponding change in other. To solve real world problem, one must equip his mind with the understanding of analytic geometry (Young, 1909). In addition, the idea of coordinate plan should be well grounded. And further, to overcome the cognitive load and to develop the proficiency of the students in this subject both synthetic and analytic approach should be used (Timmer & Verhoef, 2012).

THE VAN HIELE'S MODEL AND ANALYTIC GEOMETRY

Due to the axiomatic nature of school geometry, it is a tough cognitive process in teaching and learning. And, to learn and teach this subject with ease, two Dutch educators (Husband and Wife) Pierre Van Hiele and Dina Van Hiele-Geld developed a cognitive geometric thinking model of five discrete levels. They explained the cognitive growth of students in levels through a structured hierarchy of stages (Pandiscio & Knight, 2010). Each of these levels, constituted a definite characteristic in terms of activities and instructions. The students' progression through each level depends on the activities and instructions and its implication by teacher. What is more, the geometrical understanding of students depends on their active participation in a well-designed activity, and the proper objectives of the lesson, context of study, involvement in discussion rather than memorization; all lead to raising the levels.

In the same way, variable and parameter that always make analytic geometry abstract, although both stand for arithmetic and both have distinguished geometric behavior. In learning and uncovering the structure of analytic geometry proper timing and activity are integral. Additionally, due to lack of structural hierarchy in the thinking pattern of the structure of analytic geometry, students could not reach the formal stage. Their non- pattern thinking behavior always results in low concept. Therefore, instructions should always follow the students' thinking behavior pattern and should be intended to foster development from one level to the next (Van Hiele, 1999). The description of this geometric thinking model and application with reference to analytic geometry can be described as (Burger & Shaughnessy, 1986; Chan, Tsai, & Huang, 2006; Mason, 1998; Kospentaris & Spyrou, 2007; Pandiscio & Knight, 2010; Yazdani, 2007).

Level 1 (Visualization)

In the first stage of the model, students observe the object in gestalt, and decisions are mostly perception based rather than reasoning. And students treat the figure without its traits, definitions and descriptions. In addition, students just learn in this stage the geometric vocabulary. Similarly, in analytic geometry the concept of a function, relation or equation is the most important one. While discussing linear equation or quadratic equation students must know the object. At the first level, students must know the structure of concept. Through arithmetic and table, they know the shape of equation. For example, in understanding equations $ax + by + c = 0$ or $x^2 + y^2 = 4$

At first level, students must know that $\{ax + by + c = 0\}$ is linear equation, representing the straight line, and the other is quadratic (circle equation) representing a circle, without further description and traits.

Level 2 (Analysis)

At this level, students identify the traits of the object, figure or shape. They name and analyze the traits of objects without observing the mutual relationships between their traits. We can call this level, trait oriented level. In which students cannot define and describe the object completely. Though, the necessary and sufficient conditions for an object according to their properties are still ambiguous. However, in teaching context, first two levels of van Hiele model are very important and students should apply it in different context.

In analytic geometry, both linear and quadratic equations represent a specific figure, and it is drawn by the totality of its properties. And students should discover these properties by themselves rather be offered ready-made by the teacher. In the context of analytic geometry if we consider the linear equation or circle equation i.e. $y = mx + b$ & $x^2 + y^2 = a^2$, the instructional goals of level 2 for these two equations would be: that students must know about the distinct types and analytical attributes of these equations without their mutual relationships. Such as, in the above linear equation "(x, y)" shows a point, "m" stands for slope and "b" for y-intercept. Accordingly, to learn different attributes of a line equation, the practicality of the activity is the most important thing. Lastly, in analytic geometry, students must get the sense of line- equation in both ways: Algebraically and geometrically.

Level 3 (Abstraction)

From this level deductive geometry takes on, and it's the level where students perceive relationships between properties of figures within and among the classes. At this stage, students are capable of reasoning with meaningful description along with class inclusion. For example, students at this stage can use the transformation logic that is, "square being a type of rectangle". They can also understand and use the definitions. Nevertheless, the concept nesting is understood although intrinsic characteristics still could not be manipulated.

Likewise, with reference to the line equation, for example, $y = 2x + 1$ & $y = 2x + 2$ are two distinct parallel lines because their slopes are equal with different intercepts. Students at this stage do not know, whether two lines having different intercepts are parallel or not. In the same way, in equation of line $y = 3x + 1$, students must know the analytical relationships between the slope and y-intercept in a concrete way. Students of this level must understand different semiotic representation in the same register, with facility.

Level 4 (Formal Deduction)

At this level, students can construct proofs. In the specification of the attributes of this level, students can understand the inter-relationship between undefined terms, definitions, axioms; postulates, theorem and proof, and they can use it with facility. The student reasons formally and can look at different possibility within the context of a mathematical system. Students at this stage ask ambiguous questions and can rephrase the problem tasks into precise language. In addition to the attributes of this level, frequent conjecturing, attempts to verify conjectures deductively, systematic use of arguments, and sufficient conditions all are included and understood.

Students of this stage must know about the line equation, such as, "if the two lines are parallel", then the lines will be of same slopes. In addition to that, students at this stage must reason also, if lines are having same slopes, then the lines may be parallel or coincided. In the same way, for the axiomatic rule for two perpendicular lines, if two inclined lines are perpendicular then the product of their slopes must be equal to -1. Moreover, in equation of line $y = mx + b$ students must know the role of "m" and "b" in abstract way rather than concrete.

Level 5 (Rigor)

Students at this level understand the formal aspects of deduction, such as establishing and comparing mathematical systems. In the same way, they can understand the use of "indirect proof and proof" by contra positive and can understand non-Euclidean system. Transformation of different systems can take place and students can compare different axiomatic systems.

GEOGEBRA FEATURES REGARDING ANALYTIC GEOMETRY TEACHING

GeoGebra as a Representational Tool

Mathematical ideas and concepts are only comprehended through variety of representations and the strength of understanding relies on the functional relationships between these representations. Traditional teachings lack versatile representations and GeoGebra is the best technological tool that produces results in multiple representations. Bayazit and Aksov (2010), categorized representation into two: visible and invisible. Visible means to represent a concept in a concrete way either: symbol, graph, model, drawing or an algebraic expression. On contrary to visible, invisible related to mental manipulation on the bases of external representation. In fact, it is GeoGebra that can turn different possible invisibles representations of analytic geometry concepts into visible. Furthermore, GeoGebra has all the essential characteristics that should be for educational software. On a single click, GeoGebra turns the symbolic representation into geometric and vice versa.

GeoGebra as a Process Tool

GeoGebra gives the process of a concept or activity in a well-defined way. By using the tool of construction protocol, the whole activity can be known. The process of a concept can also be designed by using slider and dynamic tool.

GeoGebra as a Concept Development Tool

The main concern of psychology of learning and mathematical task is to create and develop concept related images in students' mind that are non-verbal. For this, dynamic geometry software is the best tool which enables students to grasp the concept by doing and acting with object in a flexible way that support relational thinking instead of instrumental thinking. The product of the concept can be seen after doing the process in different windows that help in compressing the concept (Karadag, 2009). Through this software one can draw the graph dynamically as a result different insight images of a concept evoke in a meaningful way (Tall & Sheath, 1983).

GeoGebra as a Proceptual Thinking Development Tool

One of the main objectives of mathematics teaching is to formalize concept and to involve the participants in learning process. Conceptual learning is a mental process that requires proper systematic strategy. In mathematics, proceptual thinking means the representation of an object through flexible symbol. Object, process and procept form proceptual thinking. Dynamic geometry software has the capacity through which we can represent a concept in multiple perspectives (Gray & Tall, 1994).

Research Related to GeoGebra Aided Instruction

In this technological age, everyone, including teachers and students having technology in their hands and around them but still in class the traditional teaching (minus technology). Although, various technological tools have been developed and being used to assist teaching and give scaffolding to the students understanding in different perspectives. Out of these, DGE'S (Dynamic Geometry Environment) offer fundamentally different learning environment with the facility of easy manipulation of objects. The features of GeoGebra are also very simple and straight forward in usage. The study of Erbas and Yenmez (2011) showed the significant effect of DGE'S on experimental group in achievement, interest and motivation in geometry learning. Besides this, experimental group process learning is the best way and showed effective result in retention. In another study, dynamic geometry was supported by digital photograph results in greater achievement and cause of permanence of knowledge (Gecü & Özdener, 2010). In the same way, Cakir and Yildirim (2006) observed the positive attitude of pre-service teacher towards the integrating of technology in classroom setting; and to foster the process of integrated technology in classroom the role of teachers and its attitudes are major. Likewise, Salim (2014) selected lower performance group for DGS treatment in comparison to the control group whose performance was higher. After the treatment the experimental group showed better performance than the control group in geometry. While, in the research study of (Olkun, Sinoplu, & Deryakulu, 2005), the instructions activities were designed on the bases of van Heile geometric thinking levels with the application tool of dynamic geometry, in the result the effective and distinguish creativity of the students were observed along with positive attitude towards learning. Further, the geometrical progression through stages was also achieved by the students on their own activities. As teachers use different tools for teaching learning mathematics, so they must keep GeoGebra tool in their toolkit to make mathematics learning and understanding alive.

Significance of the Study

This experimental study was conducted to find out the effect of GeoGebra aided instructions on students' mathematical achievement. This study is related to mathematics education and the study may be helpful in introducing GeoGebra aided instructions in teaching of mathematics in the education system of Pakistan. This study will also be helpful for general mathematics teacher community to modify their instructions with respect to GeoGebra aided instructions. As GeoGebra is specifically designed for high school mathematics, and more specifically for algebra, geometry and calculus, so this research may helpful for high and higher mathematics curriculum designers in Pakistan to integrate it in mathematics curriculum as supplementary tool for learning mathematics.

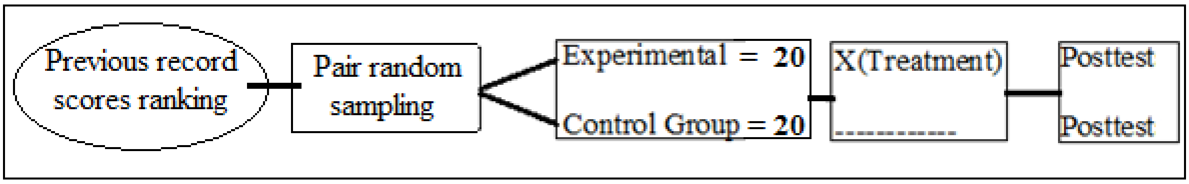

Figure 1. Research Design

HYPOTHESES OF THE STUDY

H_{O1}: Treatment does not significantly affect the mathematical achievement mean scores of the two groups.

H_{O2}: Treatment does not significantly affect the higher achievers of the two groups on the mathematical achievement mean scores.

H_{O3}: Treatment does not significantly affect the lower achievers of the two groups on the mathematical achievement mean scores.

METHOD AND TREATMENT PROCEDURE

Population

All the government and non-government higher secondary, 384207 male students, were the targeted population of this experimental study (EMIS, 2013-2014).

Sample

A total of 40 students of FG Boys Inter College Mardan Cant were selected through convenient sample.

Design of the Study

Based on previous record of the 40 selected students, the pure posttest only equivalent groups design was used. This design is very powerful and authentic in investigating the effect of experimental variable over random sampling. There can be a danger that by applying the random sampling in small sample that result can be into two non-equivalent random groups. In such cases, the effect might be owing to non-equivalent group rather than experimental variable. To reduce the risk, rank order should be used. During the matched pair-random sampling process the students were divided and exposed to two groups, on the basis of certain characteristics which should be highly correlated with the post-test (Nesteor & Schutt, 2014; Ary, Jacobs, Sorensen, & Walker, 2013; Cohen, Manion, & Morrison, 2011; Goodwin, 2010).

Sampling Procedure

There were 80 students registered in grade-12 class. Out of which a total of 40 students were selected based on their interest and willingness for this experimental study. Further, the two groups were constructed based on their previous standardized mathematics exam scores. This exam was conducted by the Board of Intermediate and Secondary Education Mardan (BISE Mardan http://www.bisemdn.edu.pk). The selected students were ranked according to their previous grade-11 exam result in mathematics subject. Their previous scores were used only for matching individuals and not for post-test. The twenty/20 matched pairs of the students were arranged, and then from each pair the one was assigned to control, and the other was assigned to experimental group.

To investigate the GeoGebra treatment effects on diverse achievers, the students of both the groups were divided into two halves: higher achievers and lower achievers. The first twelve/12students of each group were considered as higher achievers and last ten of each group was considered as lower achievers.

Instrument Development and Content Validity

In mathematics, achievement is short term cognitive ability and it is circumscribed by two main aspects that are: knowledge and skill. Both are the combination of declarative and procedural knowledge. And to measure students' mathematical achievement in this study, a test was developed around the course content of the following topics: 1) Distance formula & ratio formula 2) Slope of a line joining two points 3) Equation of a line with different formats (point slope form, two points form, slope intercept form, two intercepts form, normal form) 4) Position of a point with respect to the line 5) Parallel and perpendicular line 6) Function analysis 7) Circle equation. Further, as a good test always having good test items and writing the test items requires two dimensions: the content and the responded cognitive behavior. Based on these assumptions the mathematical achievement test of 10 MCQ'S and eight subjective questions of 100 marks were developed to measure the knowledge and skill of the students after treatment (Haladyna, 1997).

For criterion-referenced measure, the content validity is essential, and it deals with the degree of appropriateness of the test items for which the items are being assessed. In content validity the major focuses are on the instruments and the format of the test, the test score and the measurement have no significance (Wainer & Braun, 2013). And according to Farooq (2001), the content validity of a test depends on the declaration opinions of the group of experts in the relevant area. For this, the judgments of the experts indicate whether the test items really assess the desired competencies or content that is predetermined by constructor (Shroch & Coscarelli, 2007). That's why; the posttest was discussed with five mathematics educationists (Dr. Amir Zaman, Dr. Erdinç Çakıroğlu, Dr.Ayhan Kürşat ERBAŞ, Dr.Mamoon and Dr. Abdul Majeed) and the two mathematics educationist scholars (Muhammad Nasar and Murat Kol). On the bases of their experts' opinions, the posttest was modified.

Reliability of the Instrument

As all the achievement test questions are of maximum 10 marks except Q1. So, for equality in questions marks, the Q1 was converted into 10 marks and then the reliability of the whole achievement test was calculated using SPSS. The reliability of the achievement test was measured 0.92.

Treatment

See **Table 1**.

Table 1. Participants of the study

Groups	High Achievers	Low achievers	Total
GeoGebra aided instruction	12	8	20
Traditional lecture based	12	8	20
Total	24	16	40

Orientation of Experiment and Facilitator

Six-week duration of 22 lessons were constructed (available at http://tube.geogebra.org/mkhalilkhan) and conducted on 1st August 2014 and ended on 19th September 2014. Students' maturation in the subject and teacher competency, these variables influence the dependent variable. To control these threats the experiment was started in summer vacations and both the groups were taught by the same teacher under the supervision of researcher. Moreover, the only difference in the two treatment patterns was that the GeoGebra aided instructions were performed in a well-equipped computer lab for the specified content. And about ten computers were arranged there in U-shape and the students worked there in pairs. On first two days, the experimental group students were trained about GeoGebra using the two main topics: GeoGebra installation and GeoGebra user interface. While all the lessons were taught through GeoGebra applets and directed activities. The experimental group students were mostly involved in GeoGebra learning environment through drill and practice. In addition, they were engaged to learn the analytic geometry concept through applets and further they were also assigned to solve the problems with the help of GeoGebra. The teacher's role was of a facilitator

All the activities were constructed in such a manner that provided the sense and the process of concept. The underlying concept of every mathematical object was visualized dynamically.

GeoGebra Activity

The ratio formula activity was one of the 22 activities.

The focus objectives of this GeoGebra activity were:

1. The process of Ratio formula concept validated arithmetically through GeoGebra tools and visualization of its different sub concepts.
2. Through similarity of triangle using ratio concept of two sides of triangle the concept was taught.
3. Dynamic features of ratio formula concept were connected through sliders.

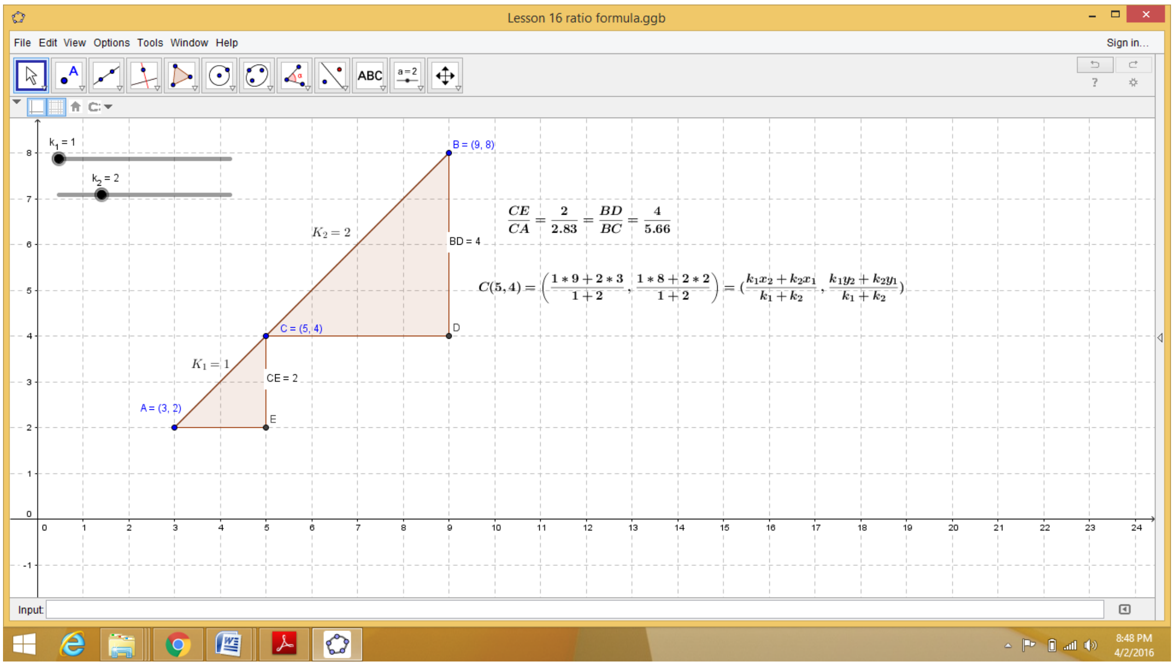

Figure 2. GeoGebra Activity

Instructional method

If a point divide two point's line segment internally in the ratio of k_1 and k_2, then the by ratio formula the coordinate of point$(x, y) = \left(\frac{k_1x_2+k_2x_1}{k_1+k_2}, \frac{k_1y_2+k_2y_1}{k_1+k_2}\right)$. Students were facilitated in performing the activity. They were assisted to get the sense of concept through formula application and validation. The basis of this formula relies on the ratio of two similar triangle comparison. By comparing the ratio to perpendicular and hypoteneous of both triangle after construction.

Assessment: Students were asked to find the point which divides the line segment in 1: 2 and the points are (1, 1) and (7, 7). Solution through ratio logic and validate and verify it through formula. Find the points that trisecting the line segment joining the points A (2, 1) B (3, 6) Also find the points that divide the line segment (2, 1) (8, 5) in different possible positive number ratio.

Data Analyses

To compare experimental and control group along with their nested high and low achievers group in their mathematical achievement test, the t-test was used.

Table 2. Significance of difference for experimental and control groups' mathematical achievement Post-test

Group	N	Mean	SD	Df	t-test for Equality of Means		
					t	Sig. (2-tailed)	Mean Diff
Exp.	20	70	13.5	38			
Cont.	20	56.95	17.5		2.6	.012	13.05

The **Table 2** of mathematical achievement for both the groups (experimental group and control group) shows that the average score obtained by the experimental group was higher than that of the control group. At the same time if we look at the column of the standard deviation, we can see that the standard deviation of the experimental group is lower than that of the control group. The comparative compression in the scores distribution of experimental group implies that GeoGebra did effect the mathematical achievement of all the participants across the experimental group as compared to control group.

In addition, in **Table 2**, $t_{\text{calculated}}$ =2.6 and p-value=0.012<0.05, so we reject the null hypothesis. Thus the treatment did significant effect on the mathematical achievement scores of experimental group. The T-test result indicates that all the participants in the experimental group (Mean=70; SD=13.5) did well in the mathematical achievement test than participants in the control group (Mean=56.95; SD=17.5).

Table 3. Between-subjects factors performance on mathematical achievement post-test

Group	N	Mean	Std. Deviation
Exp. High Achiever	12	78.33	10.79
Exp. Low. Achiever	8	57.50	3.93
Cont. High Achiever	12	66.92	15.49
Cont. Low. Achiever	8	42.00	5.37

The descriptive statistics in **Table 3**, for both the higher and lower achiever groups (experimental group and control group) shows that the average scores` obtained by the Exp.H.A group was higher than that of the Cont.H.A group. In the same way, the average marks obtained by Exp.L.A group were higher than that of the Cont.L.A group. At the same time, if we look at the column of the standard deviation, we can see that the standard deviations of the experimental groups (Exp.H.A & Exp.L.A) are lower than that of the control groups (Cont.H.A & Cont.L.A).

Table 4. Comparisons between-subjects factors in the mathematical achievement post-test

(I) Group	(J) Group	Mean Diff (I-J)	t-test for Equality of Means	
			t values	Sig. (2-tailed)
Exp.H.A	Cont.H.A	11.42	2.1	0.04
Exp.L.A	Cont.L.A	15.5	6.6	0.000

Table 4, shows that the treatment did affect the mean's scores of the diverse groups at the significance level. In conclusion, we reject HO3 and HO2, and accept the alternatives, that is, treatment did significantly affect the higher and lower achievers of Experimental groups in mathematical achievement mean scores. Below are the graphs that show the performances of experimental high and lower achievers on the mathematical achievement post-test as compared with the control high and lower achievers. The graph indicates that the performance of experimental low achiever participants was outstanding, and they improved their mathematical achievement ability at significant mark as compared to the traditional low achiever participants.

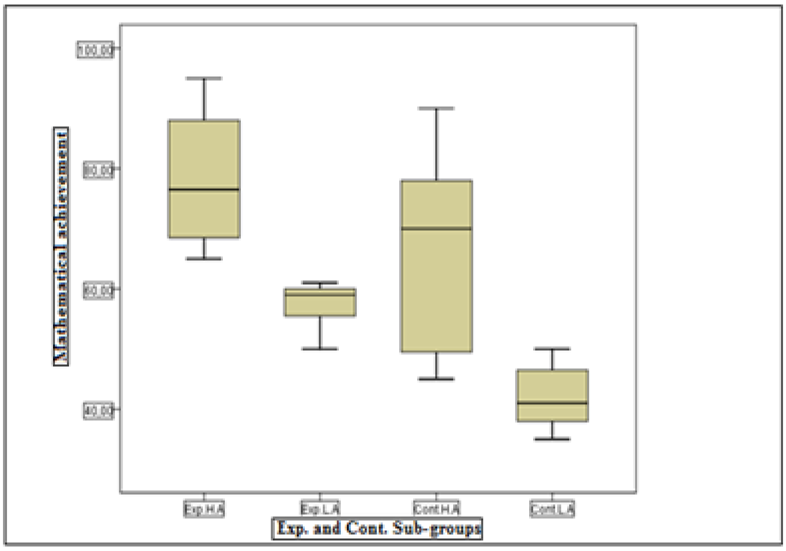

Figure 3. Box plot of the M.Ach post-test scores between-subjects factors

DISCUSSION AND CONCLUSION

The main object of the study was to discover the influential effect of GeoGebra activities aided instructions of experimental group in comparison with traditional teaching method of control group in analytic geometry. Results of the study based on post-test indicated that Geogebra aided instructions have significantly affected the mathematical achievement of the experimental group along with their nested group in comparison with tradition instruction method for control group. The mean difference of the two groups was statistically significant at 0.05 level. So, the null hypotheses were not accepted. In conclusion, GeoGebra activities significantly increased the mathematical achievement of the experimental diverse achievers in comparison to control diverse achievers. As, the standard deviation of the experimental groups are lower than control groups, this shows that the GeoGebra aided instruction is equally affect the whole class. These findings also support the claims of different studies conducted by; Mwei, Too, and Wando (2011), Tran, Nguyen, Bui, and Phan (2014), Bakar, Fauzi, and Tarmizi (2010), Demirbilek and Özkale (2014), Erbas and Yenmez (2011), and Shadaan and Leong (2013). All of them reported that the technology-integrated environments cause increase in competencies, achievement, positive attitude, mathematical reasoning and mathematical thoughts across different grades.

RECOMMENDATIONS

GeoGebra is a free package and it's specially designed for calculus and analytic geometry. Its implementation and usage is very simple and non-algorithmic. A proper computer lab is the basic requirement to implement it in educational institute. So, the school and college administration need to facilitate students and teachers with respect to computer lab. Further, teacher's specialized knowledge is necessary for teaching of each discipline of mathematics. So, for analytic geometry, the dual approach in representation as well in thinking is necessary for teaching and understanding of this subject, and for this GeoGebra is the best free easiest tool in exploring its different dual aspects through dynamic ways. Teaching and activity should be developed based on discrete stages. Through GeoGebra these stages should be concretized in dynamic ways. Teachers should be dynamic in using various instructional methods. And, to produce better result students should be engaged in exploring the concept dynamically and teacher should assist the students to construct the concept by using GeoGebra tools. Because of the effectiveness of this GeoGebra software, it should be implemented in pre-service teacher program (especially in Pakistan) for instructional setting.

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APPENDIX

	Exp. Group	Cont. Group
Stds	Grade-11 Previous math test scores Mark Pretest scores	Grade-11 Previous math scores
S_1	84	84
S_2	83	80
S_3	77	77
S_4	76	76
S_5	74	74
S_6	74	73
S_7	72	72
S_8	70	70
S_9	69	69
S_{10}	68	67
S_{11}	66	66
S_{12}	64	65
S_{13}	58	58
S_{14}	56	56
S_{15}	55	56
S_{16}	53	54
S_{17}	53	53
S_{18}	52	53
S_{19}	49	50
S_{20}	33	43

	Posttest Score	Stds	Posttest Score
Stds	Exp. Group		Control Group
1	95	1	90
2	84	2	78
3	91	3	85
4	92	4	75
5	65	5	70
6	85	6	78
7	70	7	65
8	72	8	50
9	75	9	70
10	78	10	45
11	67	11	49
12	66	12	48
13	61	13	50
14	60	14	42
15	58	15	48
16	60	16	40
17	58	17	45
18	60	18	40
19	50	19	36
20	53	20	35

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